

# $\alpha$ decay half-lives of new superheavy nuclei within a generalized liquid drop model

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The  $\alpha$  decay half-lives of the recently produced isotopes of the 112, 114, 116 and 118 nuclei and decay products have been calculated in the quasi-molecular shape path using the experimental  $Q_\alpha$  value and a Generalized Liquid Drop Model including the proximity effects between nucleons in the neck or the gap between the nascent fragments. Reasonable estimates are obtained for the observed  $\alpha$  decay half-lives. The results are compared with calculations using the Density-Dependent M3Y effective interaction and the Viola-Seaborg-Sobiczewski formulae. Generalized Liquid Drop Model predictions are provided for the  $\alpha$  decay half-lives of other superheavy nuclei using the Finite Range Droplet Model  $Q_\alpha$  and compared with the values derived from the VSS formulae.

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The synthesis of superheavy elements has advanced using both cold [1] and warm fusion reactions. Recently [2, 3, 4], isotopes of the elements 112, 114, 116 and 118 have been produced in fusion-evaporation reactions at low excitation energies by irradiations of the  $^{233,238}\text{U}$ ,  $^{242}\text{Pu}$ ,  $^{248}\text{Cm}$  and  $^{249}\text{Cf}$  targets with  $^{48}\text{Ca}$  beams. The main decay mode is the  $\alpha$  emission and the  $\alpha$  decay energies and half-lives of fourteen new  $\alpha$  decaying nuclei have been measured. Some questions have been raised [5] about these superheavy element findings. In similar sophisticated experiments at other places [6, 7] the  $\alpha$  cascades were not observed.

The pure Coulomb barrier sharply peaked at the touching point does not allow to determine correctly the partial  $\alpha$  decay half-lives. It is probable that the  $\alpha$  decay takes place in the quasi-molecular shape path where the nucleon-nucleon forces act strongly during the formation of the neck between the nascent fragments and after the separation and a proximity energy term must be added in the usual development of the liquid-drop model [8]. The generalized liquid drop model (GLDM) which includes such a proximity energy term has allowed to describe the fusion [9], fission [10], light nucleus [11] and  $\alpha$  emission [12, 13] processes.

The purpose of this work is to determine the partial  $\alpha$  decay half-lives of these superheavy elements within this GLDM from the experimental  $Q_\alpha$  values using the WKB approximation and to compare with the experimental data and the calculations with the Density-Dependent M3Y (DDM3Y) effective interaction [14] and the Viola-Seaborg formulae with Sobiczewski constants (VSS) [15]. Finally predictions within the GLDM and VSS formulae are given for the partial  $\alpha$  decay half-lives of the still non observed superheavy nuclei ranging from Sg to  $Z = 120$ .

For a deformed nucleus, the macroscopic GLDM en-

ergy is defined as [9].

$$E = E_V + E_S + E_C + E_{\text{Rot}} + E_{\text{Prox}}. \quad (1)$$

When the nuclei are separated:

$$E_V = -15.494 [(1 - 1.8I_1^2)A_1 + (1 - 1.8I_2^2)A_2] \text{ MeV}, \quad (2)$$

$$E_S = 17.9439 [(1 - 2.6I_1^2)A_1^{2/3} + (1 - 2.6I_2^2)A_2^{2/3}] \text{ MeV}, \quad (3)$$

$$E_C = 0.6e^2 Z_1^2/R_1 + 0.6e^2 Z_2^2/R_2 + e^2 Z_1 Z_2/r, \quad (4)$$

where  $A_i$ ,  $Z_i$ ,  $R_i$  and  $I_i$  are the mass number, charge number, radii and relative neutron excesses of the two nuclei.  $r$  is the distance between the mass centres. The radii  $R_i$  are given by [8]:

$$R_i = (1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}) \text{ fm}. \quad (5)$$

This formula allows to follow the experimentally observed increase of the ratio  $r_i = R_i/A_i^{1/3}$  with the mass; for example,  $r_0 = 1.13 \text{ fm}$  for  $^{48}\text{Ca}$  and  $r_0 = 1.18 \text{ fm}$  for  $^{248}\text{Cm}$ .

For one-body shapes, the surface and Coulomb energies are defined as:

$$E_S = 17.9439(1 - 2.6I^2)A^{2/3}(S/4\pi R_0^2) \text{ MeV}, \quad (6)$$

$$E_C = 0.6e^2(Z^2/R_0) \times 0.5 \int (V(\theta)/V_0)(R(\theta)/R_0)^3 \sin \theta d\theta. \quad (7)$$

$S$  is the surface of the one-body deformed nucleus.  $V(\theta)$  is the electrostatic potential at the surface and  $V_0$  the surface potential of the sphere.

The rotational energy is determined within the rigid-body ansatz:  $E_{\text{Rot}} = \frac{\hbar^2 l(l+1)}{2I_\perp}$ . The surface energy results

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from the effects of the surface tension forces in a half space. When there are nucleons in regard in a neck or a gap between separated fragments an additional term called proximity energy must be added to take into account the effects of the nuclear forces between the close surfaces. This term is essential to describe smoothly the one-body to two-body transition and to obtain reasonable fusion barrier heights. It moves the barrier top to an external position and strongly decreases the pure Coulomb barrier.

$$E_{\text{prox}}(r) = 2\gamma \int_{h_{\min}}^{h_{\max}} \Phi [D(r, h)/b] 2\pi h dh, \quad (8)$$

where  $h$  is the distance varying from the neck radius or zero to the height of the neck border.  $D$  is the distance between the surfaces in regard and  $b = 0.99$  fm the surface width.  $\Phi$  is the proximity function of Feldmeier [16]. The surface parameter  $\gamma$  is the geometric mean between the surface parameters of the two nuclei or fragments. The combination of the GLDM and of a quasi-molecular shape sequence has allowed to reproduce the fusion barrier heights and radii, the fission and the  $\alpha$  and cluster radioactivity data.

For the  $\alpha$  emission this very accurate formula simulates the proximity energy [17]:

$$E_{\text{prox}}(r) = (4\pi\gamma)e^{-1.38(r-R_\alpha-R_d)} [0.6584A^{2/3} - (\frac{0.172}{A^{1/3}} + 0.4692A^{1/3})r - 0.02548A^{1/3}r^2 + 0.01762r^3]. \quad (9)$$

To obtain the  $\alpha$  decay barrier from the contact point between the nascent  $\alpha$  particle and daughter nucleus it is sufficient to add this proximity energy to the Coulomb repulsion.

The half-life of a parent nucleus decaying via  $\alpha$  emission is calculated using the WKB barrier penetration probability. In a unified fission model, the decay constant of the  $\alpha$  emitter is simply defined as  $\lambda = \nu_0 P$ . The assault frequency  $\nu_0$  has been taken as  $\nu_0 = 10^{20} \text{s}^{-1}$ . The barrier penetrability  $P$  is calculated within the action integral

$$P = \exp[-\frac{2}{\hbar} \int_{R_{\text{in}}}^{R_{\text{out}}} \sqrt{2B(r)(E(r) - E(\text{sphere}))}]. \quad (10)$$

The deformation energy (relative to sphere) is small until the rupture point between the fragments [12] and the two following approximations may be used:  $R_{\text{in}} = R_d + R_\alpha$  and  $B(r) = \mu$  where  $\mu$  is the reduced mass.  $R_{\text{out}}$  is simply  $e^2 Z_d Z_\alpha / Q_\alpha$ . The partial half-life is related to the decay constant  $\lambda$  by  $T_{1/2} = \frac{\ln 2}{\lambda}$ . The  $\alpha$  decay half-lives of the recently produced superheavy nuclei calculated with the GLDM using the experimental  $Q_\alpha$  value and without considering the rotational contribution are presented in Table 1. The results agree reasonably with the experimental data indicating that a GLDM taking account the proximity effects, the mass asymmetry, and

an accurate nuclear radius is sufficient to reproduce the  $\alpha$  decay potential barriers when the experimental  $Q_\alpha$  value is known. The results obtained with the DDM3Y interaction agree with the experimental data as the GLDM predictions and largely better than the VSS calculations. This shows that a double folding potential obtained using M3Y [18] effective interaction supplemented by a zero-range potential for the single-nucleon exchange is very appropriate because its microscopic nature includes many nuclear features, in particular a potential energy surface is inherently embedded in this description. This double agreement shows that the experimental data themselves seem to be consistent. For most nuclei the predictions of the VSS model largely overestimate the half lives. The blocking effect is probably treated too roughly.

The half live of  $^{294}118$  is slightly underestimated in the three theoretical calculations possibly due to the neutron submagic number  $N = 176$ . In Ref. [19], it is also pointed out that for oblate deformed chain of  $Z = 112$ , the shell closure appears at  $N = 176$ .

Most of the theoretical half lives using GLDM are slightly smaller than the experimental data. A reason is perhaps that the rotation of the nuclei is neglected in the present calculations. The term  $\hbar^2 l(l+1)/(2I_\perp)$  in Eq.(??) represents an additional centrifugal contribution to the barrier which reduces the tunnelling probability and increases the half lives. A second reason is that the shell effects and pairing correlation are not explicitly included in the alpha decay barrier, in spite of their global inclusion in the decay energy  $Q$ .

The experimental  $\alpha$  decay half-lives are between the close theoretical values given by the GLDM and the ones derived from the VSS formulae. Thus predictions of the  $\alpha$  decay half lives with the GLDM and VSS formulae are possible. In the next calculations the experiential  $Q_\alpha$  values are taken from the FRDM [20] which reproduces all known experimental data of ground state properties of a large number of nuclei and gives good predictions for nuclei far from the  $\beta$  stability line and the superheavy nucleus region. In Ref. [13]  $T_{1/2}$  obtained using the  $Q_\alpha$  values given by the Thomas-Fermi model can be found.

$\alpha$  decay half-lives for  $Z = 106$  to  $Z = 120$  isotopes are shown in Fig.1, the open dots indicating the results of GLDM and the black triangles the ones derived from the VSS formulae.

The half-life values vary from years to microseconds. A narrow window exists between the two predictions for each isotope and the unknown  $\alpha$  decay half-lives of SHN may lie in this window assuming that the FRDM  $Q_\alpha$  is correct.

The FRDM  $Q_\alpha$  value explicitly displays a minimum at the submagic number  $N = 162$ . It is progressively eroded by the neutron deficiency and the subshell disappears completely from  $Z = 115$ . This induces a small first hump in the predicted  $\log_{10}[T_{1/2}(s)]$  curves till  $Z = 114$ . Before this first submagic number the half-lives increases rapidly with  $A$  from Db to  $Z = 114$  isotopes. For the results of

TABLE I: Comparison between experimental  $\alpha$  decay half-lives [4] and results obtained with the GLDM, the DDM3Y effective interaction [14] and the VSS formulae.

Parent $Z$	Nuclei $A$	Expt. $Q(\text{MeV})$	Expt. $T_{1/2}$	DDM3Y $T_{1/2}$	GLDM $T_{1/2}$	VSS $T_{1/2}$
118	294	$11.81 \pm 0.06$	$1.8^{+15}_{-1.3}$ ms	$0.66^{+0.23}_{-0.18}$ ms	$0.15^{+0.05}_{-0.04}$ ms	$0.64^{+0.24}_{-0.18}$ ms
116	293	$10.67 \pm 0.06$	$53^{+62}_{-19}$ ms	$206^{+90}_{-61}$ ms	$22.81^{+10.22}_{-7.06}$ ms	$1258^{+557}_{-384}$ ms
116	292	$10.80 \pm 0.07$	$18^{+16}_{-6}$ ms	$39^{+20}_{-13}$ ms	$10.45^{+5.65}_{-3.45}$ ms	$49^{+26}_{-16}$ ms
116	291	$10.89 \pm 0.07$	$6.3^{+11.6}_{-2.5}$ ms	$60.4^{+30.2}_{-20.1}$ ms	$6.35^{+3.15}_{-2.08}$ ms	$336.4^{+173.1}_{-113.4}$ ms
116	290	$11.00 \pm 0.08$	$15^{+26}_{-6}$ ms	$13.4^{+7.7}_{-5.2}$ ms	$3.47^{+1.99}_{-1.26}$ ms	$15.2^{+9.0}_{-5.6}$ ms
114	289	$9.96 \pm 0.06$	$2.7^{+1.4}_{-0.7}$ s	$3.8^{+1.8}_{-1.2}$ s	$0.52^{+0.25}_{-0.17}$ s	$26.7^{+13.1}_{-8.7}$ s
114	288	$10.09 \pm 0.07$	$0.8^{+0.32}_{-0.18}$ s	$0.67^{+0.37}_{-0.27}$ s	$0.22^{+0.12}_{-0.08}$ s	$0.98^{+0.56}_{-0.40}$ s
114	287	$10.16 \pm 0.06$	$0.51^{+0.18}_{-0.10}$ s	$1.13^{+0.52}_{-0.40}$ s	$0.16^{+0.08}_{-0.05}$ s	$7.24^{+3.43}_{-2.61}$ s
114	286	$10.35 \pm 0.06$	$0.16^{+0.07}_{-0.03}$ s	$0.14^{+0.06}_{-0.04}$ s	$0.05^{+0.02}_{-0.02}$ s	$0.19^{+0.08}_{-0.06}$ s
112	285	$9.29 \pm 0.06$	$34^{+17}_{-9}$ s	$75^{+41}_{-26}$ s	$13.22^{+7.25}_{-4.64}$ s	$592^{+323}_{-207}$ s
112	283	$9.67 \pm 0.06$	$4.0^{+1.3}_{-0.7}$ s	$5.9^{+2.9}_{-2.0}$ s	$0.95^{+0.48}_{-0.32}$ s	$41.3^{+20.9}_{-13.8}$ s
110	279	$9.84 \pm 0.06$	$0.18^{+0.05}_{-0.03}$ s	$0.40^{+0.18}_{-0.13}$ s	$0.08^{+0.04}_{-0.02}$ s	$2.92^{+1.4}_{-0.94}$ s
108	275	$9.44 \pm 0.07$	$0.15^{+0.27}_{-0.06}$ s	$1.09^{+0.73}_{-0.40}$ s	$0.27^{+0.16}_{-0.10}$ s	$8.98^{+5.49}_{-3.38}$ s
106	271	$8.65 \pm 0.08$	$2.4^{+4.3}_{-1.0}$ min	$1.0^{+0.8}_{-0.5}$ min	$0.33^{+0.28}_{-0.16}$ min	$8.6^{+7.3}_{-3.9}$ min

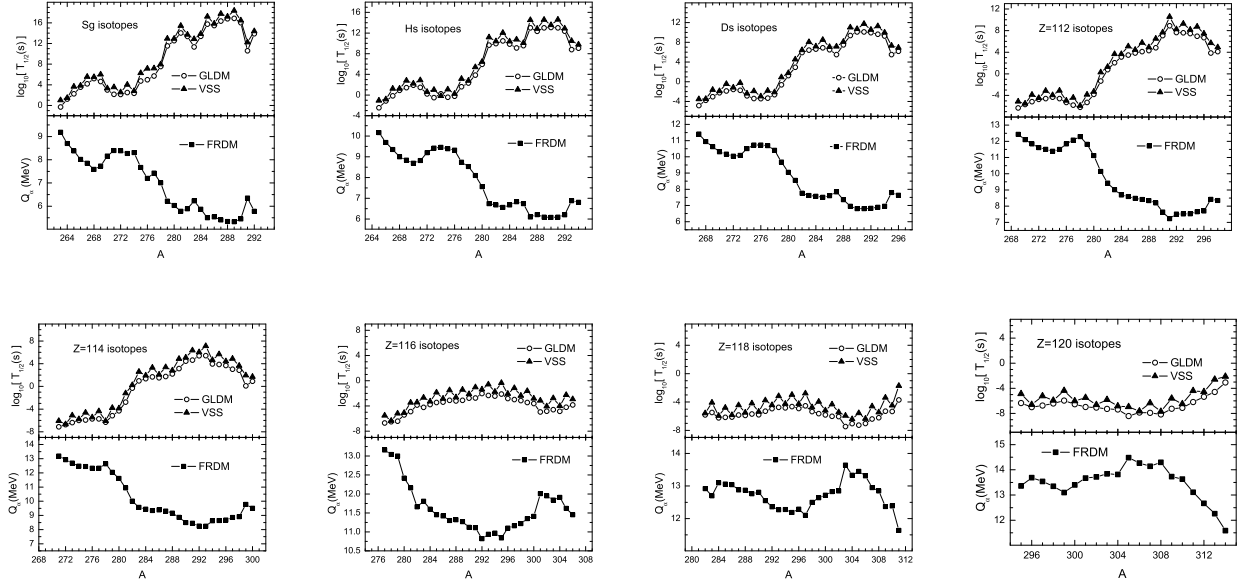


FIG. 1: Comparison between calculated  $\alpha$ -decay half-lives of Sg, Hs, Ds, 112, 114, 116, 118 and 120 isotopes using the GLDM and the VSS formulae.

VSS formulae, it seems that the turning point is delayed a little and appears at neutron number  $N=163$ , for the blocking effect has been magnified and the half lives are overestimated.  $N=184$  is always a closure shell. Subclosure or closure shells exist also around  $N=176$  for  $Z=105$  to  $Z=112$ . The most stable nuclei should stand about  $N=184$  when the proton number is higher than  $Z=115$ , but stand around  $N=176$  for lower  $Z$  values.

As a conclusion the half-lives for  $\alpha$ -radioactivity have been analyzed in the quasimolecular shape path within a Generalized Liquid Drop Model including the proximity effects between nucleons and the mass and charge asymmetry. The results are in reasonable agreement with the

published experimental data for the alpha decay half-lives of isotopes of charge 112, 114, 116 and 118 and close to the ones derived from the DDM3Y effective interaction. The experimental  $\alpha$  decay half lives stand between the GLDM calculations and VSS formulae results and the  $\alpha$  decay half-lives of still non-observed superheavy nuclei have been predicted within the GLDM and VSS approaches and  $Q_\alpha$  derived from the FRDM.

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